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1997 J. Phys.: Condens. Matter 9 9299

(http://iopscience.iop.org/0953-8984/9/43/014)

J. Phys.: Condens. Matter **9** (1997) 9299–9313. Printed in the UK PII: S0953-8984(97)83654-5

Thermal properties of a two-dimensional electron gas under a one-dimensional periodic magnetic field

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Received 24 April 1997, in final form 21 July 1997

Abstract. In the regime of linear response theory, a detailed investigation has been made of the thermodynamic and thermotransport properties of a two-dimensional electron gas modulated by a one-dimensional periodic magnetic field. The presence of the magnetic modulation leads to Weiss-like oscillations in the thermal magnetotransport coefficients. At lower temperatures ($T = 1, 2$ K) and for a weak magnetic field ($B_0 < 0.2$ T), there are just Weiss oscillations, while at $B_0 > 0.2$ T, the usual Shubnikov–de Haas oscillations appear and overlap the envelopes of the slow oscillations (Weiss oscillations). At higher temperatures ($T = 4, 10$ K), there are just Weiss oscillations. The thermal magnetotransport coefficients ($\kappa_{\mu\nu}, (\kappa^{-1})_{\mu\nu}$) have a stronger dependence on the temperature than the electrical ones ($\sigma_{\mu\nu}, \rho_{\mu\nu}$).

1. Introduction

Since the discovery of the oscillations in magnetoresistance of the two-dimensional electron gas (2DEG) modulated by a one-dimensional (1D) weak periodic potential [1], much attention has been attracted to the study of these novel oscillations (also called Weiss oscillations) [2–14]. In a perpendicular magnetic field with a weak 1D potential modulation, the Weiss oscillations in the magnetoresistance tensor $\rho_{\mu\nu}$ are periodic in $1/B$ with a larger period than that of the Shubnikov–de Haas (SdH) oscillations. The period of the Weiss oscillations depends on both the modulation period a and the square root of the areal electron density of the 2DEG ($\sqrt{n_e}$), which contrasts with the linear dependence (on n_e) of the SdH oscillations. The amplitude of the Weiss oscillations has a weaker dependence on the temperature than that of the SdH oscillations [1, 9].

Another system of great interest is the 2DEG modulated by a 1D periodic magnetic field. In the regime of linear response theory [15], the electrical transport properties of this system have been studied by several authors, and some important results have been obtained [16–22]. The results state that, in this new system, Weiss-like oscillations occur, related to the modulation of the magnetic field, which is similar to the case for the potential-modulated system [1, 9].

As is known, thermal magnetotransport is a very important aspect for a 2DEG [9]. In this paper we wish to perform calculations on the thermodynamics and the thermal magnetotransport properties of a 2DEG for the magnetic modulation case similar to those carried out in reference [9] for the potential modulation. In order to make a comparison

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0953-8984/97/439299+15$19.50

with the results for the potential modulation case [9], we take the same modulation period and modulation strength as in reference [9] and assume that the magnetic modulation is in phase with that of the potential case [9]. We will make a detailed investigation of the Fermi energy (E_F), the magnetization (M), the susceptibility (χ), the specific heat (C), the thermopower (S), the thermal conductivity (κ), and the thermal resistance (κ^{-1}) of a 2DEG modulated by a 1D periodic magnetic field.

We organize the paper as follows. The thermodynamics of the 2DEG is studied in section 2. The thermal magnetotransport coefficients and a numerical analysis are presented in section 3. Finally, our conclusions are given and some remarks are made in section 4.

2. Thermodynamics

For a uniform magnetic field, a detailed study of the magnetization, the specific heat, the magnetothermal effect, and the thermopower of a 2DEG was made by Zawadzki and Lassnig [23]. In this section we will generalize the study to the case of a 2DEG modulated by a 1D periodic magnetic field. The discussion will be limited to a relatively low magnetic field, i.e. $B_0 < 1.0$ T. For the sake of simplicity, we do not consider spin splitting in the following discussion.

Now we consider a 2DEG lying in the (x, y) plane with a lateral weak periodically modulated magnetic field (the modulation being taken to be along the x-direction) $\boldsymbol{B} = (B_0 + B_1(x))\hat{e}_z$, where the modulation amplitude $|B_1| \ll B_0$. Using the Landau gauge for the vector potential, we take $\boldsymbol{A} = \boldsymbol{A}_0 + \boldsymbol{A}_1 = (0, A_0(x) + A_1(x), 0)$, where

$$A_0(x) = B_0 x \qquad A_1(x) = \sum_{p=1}^{\infty} 2\,\mathrm{Re}\left[\frac{B_1}{2\mathrm{i} f(g_p)} \mathrm{e}^{\mathrm{i} g_p x}\right] \qquad g_p = p\frac{2\pi}{a}.$$

$f(g_p)$ has the same dimensions as g_1 $(=2\pi/a)$, and $f(g_1) = 2\pi/a$. The energy spectrum of an electron in a non-interacting 2DEG is [22]

$$\begin{cases} E_{n,k_y} = \left(n + \dfrac{1}{2}\right)\hbar\omega_c + \displaystyle\sum_{p=1}^{\infty} \varepsilon_{n,p} \cos(pKx_0) \\ \varepsilon_{n,p} = \dfrac{1}{2}\hbar\omega_1 \left[\dfrac{pK}{f(pK)}\right] \mathrm{e}^{-p^2 u/2} [L_n^1(p^2 u) + L_{n-1}^1(p^2 u)] \end{cases} \tag{1}$$

where $\omega_c = eB_0/m^*$, $\omega_1 = eB_1/m^*$, $u = \frac{1}{2}K^2 l^2$, $K = 2\pi/a$, and $l^2 = \hbar/m^*\omega_c$. $x_0 = k_y l^2$, where k_y is the wavevector of the electron in the y-direction. In the above formula, the spin splitting is not taken into consideration.

Without loss of generality, we only consider the contribution of the lowest order of the Fourier transformation of the modulation field in the following discussion, that is

$$\boldsymbol{B} = \left(B_0 + B_1 \cos\frac{2\pi}{a} x\right)\hat{e}_z$$

which is in phase with the potential modulation [9]. In this case, the bandwidth of the nth Landau subband is $2|\varepsilon_{n,1}|$.

The Fermi energy is an important parameter for a 2DEG, and can determined from the following relation:

$$n_e = 2\int_0^{\infty} \mathrm{d}E\ f(E) D(E) \tag{2}$$

where n_e is the electron areal density. $f(E) = 1/[\mathrm{e}^{(\beta(E-E_F)} + 1]$ is the Fermi–Dirac distribution function of the energy (E), and $\beta = 1/k_B T$. E_F is the Fermi energy. The factor 2 is the spin degeneracy. $D(E)$ is the density of states (DOS) of the electron in the 2DEG [22]. In the absence of a magnetic field, the DOS is a constant, and the Fermi energy of the 2DEG is only dependent on the temperature [23]. In a perpendicular uniform magnetic field ($\boldsymbol{B}_0$), the DOS is $D(E) = (1/2\pi l^2)\sum_n \delta(E - E_n)$ [9], where l and ω_c are respectively the cyclotron radius and frequency of the electron, as defined above. The Fermi energy is an oscillatory function of the magnetic field [23]—that is, the usual SdH oscillation. As the temperature becomes higher, the amplitude of this SdH oscillation tends towards zero [23].

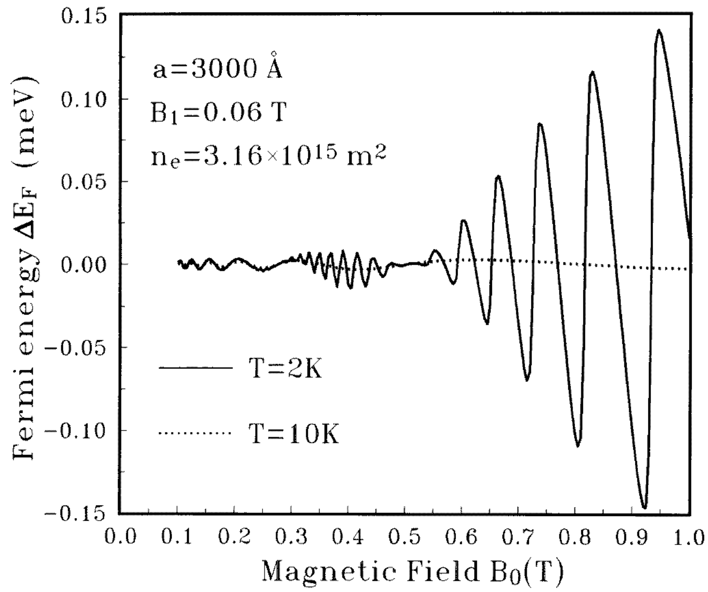


Figure 1. The change of the Fermi energy due to the 1D periodic magnetic modulation as a function of the magnetic field B_0 for $T = 2$ and 10 K. $\Delta E_F = E_F(B_1) - E_F(B_1 = 0)$.

With a one-dimensional modulating magnetic field applied, the DOS of the 2DEG is [22]

$$D(E)/D_0 = \frac{\hbar\omega_c}{2\pi}\sum_{n=0}^{\infty}\int_0^{\pi} \mathrm{d}t\ \delta(E - E_{n,t}) \tag{3}$$

where $D_0 = m^*/\pi\hbar^2$ is the DOS of a 2DEG at $B_0 = 0$. $t = (2\pi/a)x_0$, $x_0 = k_y l^2$, and $E_{n,t}$ is determined by equation (1). With this equation, equation (2) becomes

$$n_e\pi l^2 = \sum_{n=0}^{\infty}\frac{1}{\pi}\int_0^{\pi} \mathrm{d}t\ f(E)\bigg|_{E=E_{n,t}}. \tag{4}$$

Because of the occurrence of new structures in the DOS [22], there would be new oscillations in the Fermi energy of the 2DEG. The numerical results for the Fermi energy, and the parameters used, are shown in figure 1. We take the effective mass to be $m^* = 0.067m$ (in GaAs), where m is the bare mass of the electron. In figure 1 we have reset the Fermi energy according to the Fermi energy at $B = B_0$, i.e., $\Delta E_F = E_F(B_1) - E_F(B_1 = 0)$. The solid line represents the Fermi energy at temperature $T = 2$ K and the tiny-dashed one represents that at $T = 10$ K. From figure 1 we note the following findings.

(1) At lower temperatures ($T = 2$ K), there are two kinds of oscillation, the usual SdH oscillations and the Weiss-like oscillations. These latter novel oscillations are correlated

with the modulation of the applied magnetic field. The period of the Weiss oscillations is much larger than that of the SdH oscillations. At $B_0 < 0.3$ T, only Weiss oscillations are observed in ΔE_F since the SdH oscillations are too weak to be resolved. The amplitude of the Weiss oscillations is less than 0.01 meV, which is of order 0.1% of the Fermi energy at zero temperature. At $B_0 > 0.3$ T, SdH oscillations overlap the envelopes of the slow oscillations (Weiss oscillations).

(2) At higher temperatures ($T = 10$ K), the SdH oscillations are all damped out, and only the Weiss oscillations remain. Weiss oscillations in the Fermi energy have a much weaker dependence on the temperature, contrasting with the sensitive dependence of SdH oscillations.

(3) As one would expect, the zero points of ΔE_F correspond to those of the bandwidth of the Landau subband at the Fermi level (see figure 3.1 in reference [22]). One period in the bandwidth oscillation corresponds to one period of the Weiss oscillation in the Fermi energy ΔE_F. Because ΔE_F oscillates around zero and the bandwidth is always positive, we have the remarkable result that ΔE_F is zero when the bandwidth has its maxima. Therefore the Weiss oscillations in the Fermi energy stem from the oscillation properties of the Landau bandwidth of the 2DEG in a modulated magnetic field.

From a comparison with the results in reference [9], we know that (a) ΔE_F has a much smaller amplitude of Weiss oscillation than that in the case of the potential modulation; (b) the Weiss oscillations are about 90° out of phase in both cases; and (c) the Weiss oscillation in ΔE_F has a weaker dependence on the temperature than for the potential modulation case (see also figure 4 in [9]).

It is known that all of the thermodynamic properties of a system can be obtained as derivatives of the free energy of the system. For a non-interacting 2DEG, the free energy per unit area is [23, 24]

$$F = n_e E_F - 2k_B T \int_0^\infty \mathrm{d}E \; D(E) \ln(1 + \mathrm{e}^{(E_F - E)/k_B T}) \tag{5}$$

where the factor 2 is the spin degeneracy. From the free energy we could determine the electronic contribution of a 2DEG to the magnetization $M = -\partial F/\partial B_0$, the susceptibility $\chi = -\partial^2 F/\partial B_0^2$, and the specific heat $C = -T\,\partial^2 F/\partial T^2$. Because of the dependence of the free energy on the the DOS of a 2DEG, the properties of the DOS will be reflected indirectly in the different thermodynamic quantities. For the sake of simplicity, we do not consider the contribution of the scattering impurities here. From equations (3) and (5), the free energy per unit area per electron is given by

$$F/n_e = E_F - \frac{k_B T}{n_e \pi l^2} \sum_{n=0}^{\infty} \frac{1}{\pi} \int_0^\pi \mathrm{d}t \; \ln(1 + \mathrm{e}^{(E_F - E_{n,t})/k_B T}) \tag{6}$$

where $E_{n,t}$ is the energy spectrum of the electron in a 2DEG in a modulated magnetic field, determined by equation (1). The numerical results for M, χ, and C are shown in figure 2 and figure 3. Some of the parameters used are listed in the figures. Note that the thermodynamic quantities have been rescaled in the two figures. The magnetization (figure 2(a)) and susceptibility (figure 2(b)) are rescaled by the factor $M_0 = \chi_0 = n_e \mu_B^*$, where $\mu_B^* = e\hbar/2m^* \sim 0.87$ meV T^{-1} (in GaAs, $m^* = 0.067m$) is the effective Bohr magneton. Therefore M/M_0 is the magnetization per electron in units of μ_B^*, and χ/χ_0 the susceptibility per electron in units of μ_B^* T^{-1}. Figures 3(a) and 3(b) show the specific heat per electron in units of k_B. In figure 3(b), the solid line represents the specific heat at temperature $T = 2$ K (corresponding to the left-hand Y-axis) and the tiny-dashed one the specific heat at $T = 4$ K (corresponding to the right-hand Y-axis).

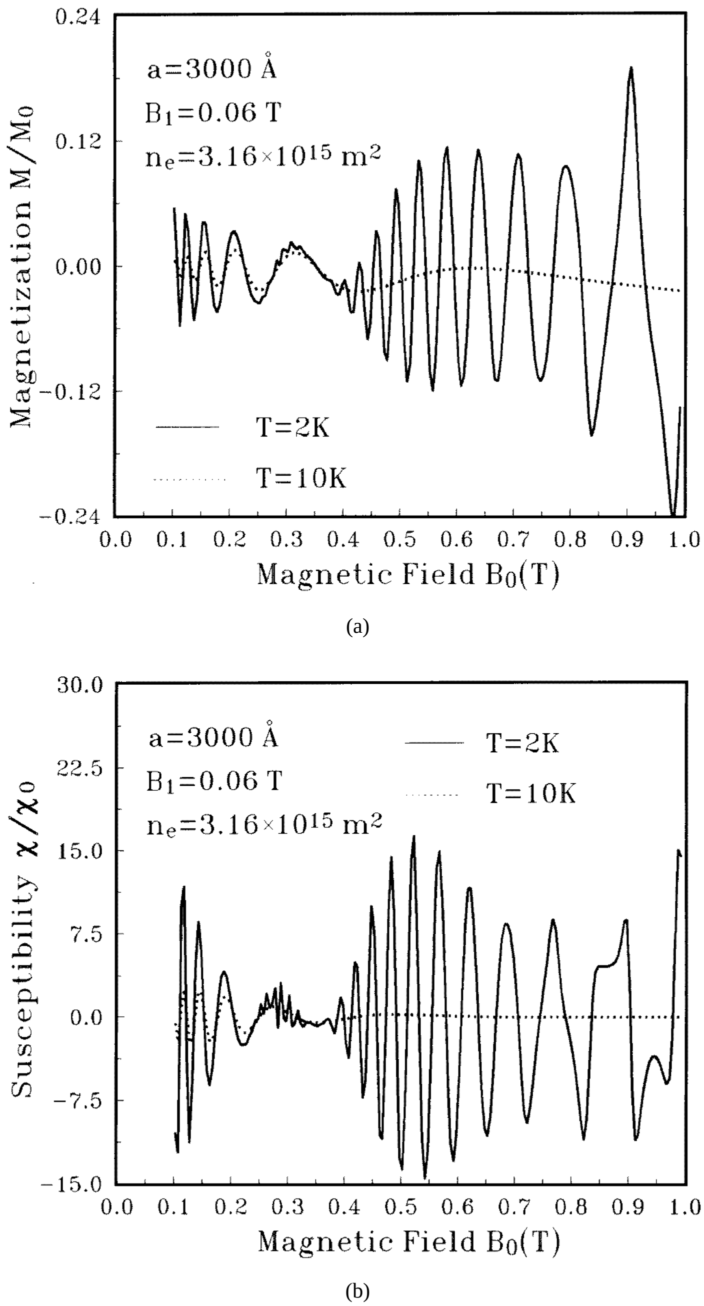
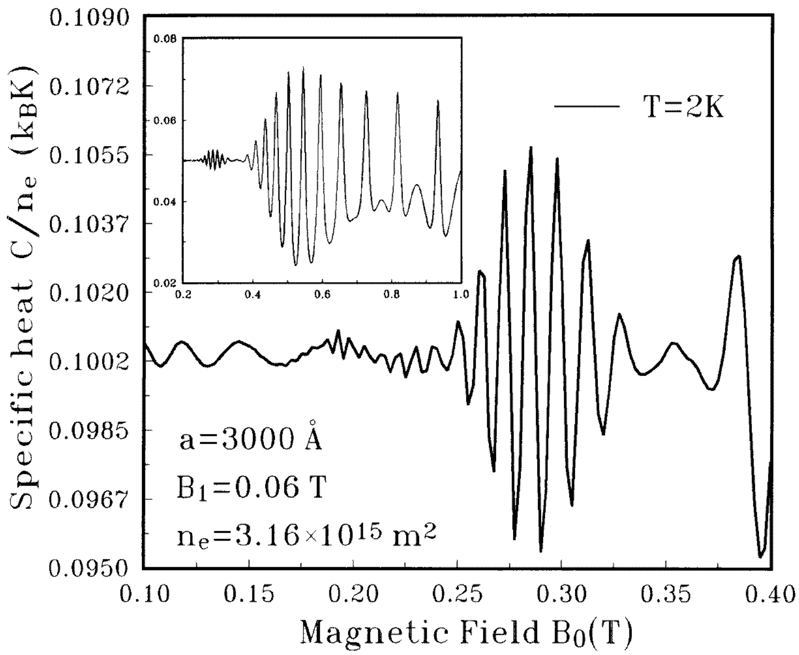


Figure 2. (a) The magnetization and (b) susceptibility due to the 1D periodic magnetic modulation as a function of the magnetic field B_0 for $T = 2$ and 10 K.

Figure 3(c) shows the change in the specific heat due to the modulating magnetic field, i.e., $\Delta C = C(B_1) - C(B_1 = 0)$. The $T\,\Delta C$–B_0 relations are plotted so as to show the specific heat at different temperatures in the same figure.

From figure 2 and 3 we note the following findings.



(a)

Figure 3. The specific heat in a magnetic field with a weak 1D periodic modulation (a) at $T = 2$ K (the inset represents the oscillations in the region $B_0 = 0.1$–1.0 T), and (b) at two temperatures $T = 2$ and 4 K, and (c) the change in the specific heat multiplied by the temperature due to the 1D modulation.

(1) All of the thermodynamic quantities exhibit two kinds of oscillation, the Weiss oscillations and the SdH ones. For weaker magnetic fields, there are only Weiss oscillations, while at higher magnetic field, the SdH oscillations appear.

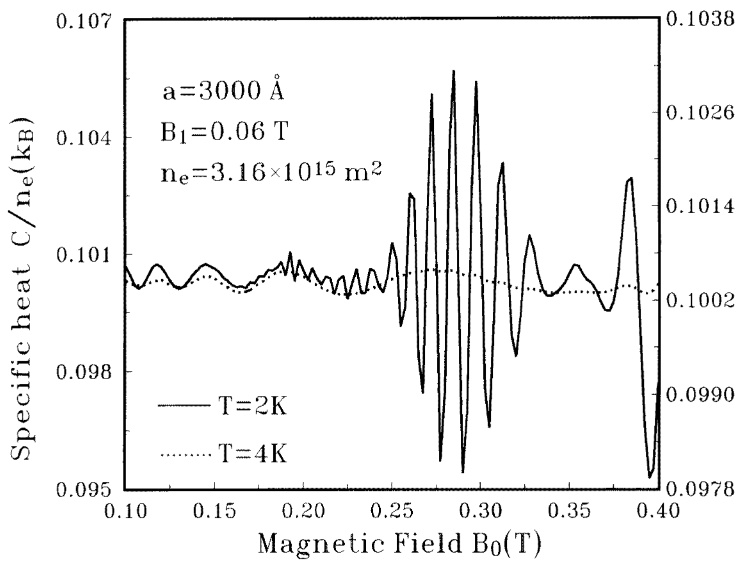
(2) The Weiss oscillations have a weaker dependence on the temperature than the SdH ones.

(3) The Weiss oscillations in the magnetization and ΔE_F are in phase, while the corresponding SdH oscillations are 180° out of phase (see also figure 1).

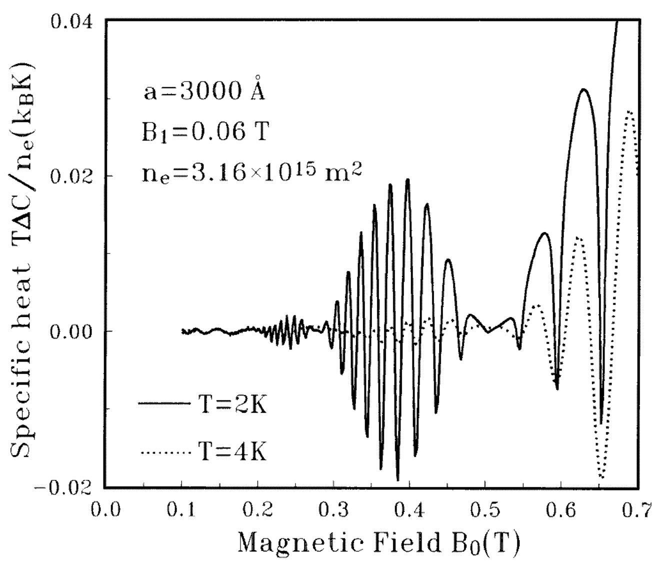
(4) The Weiss oscillations and SdH oscillations in the susceptibility and ΔE_F are 90° out of phase (see also figure 1).

(5) At lower temperature, the SdH oscillations modulating the Weiss oscillations are more obvious for the specific heat, as shown in figures 3(a) and 3(b). The Weiss oscillations in ΔC are in phase with the oscillations in the bandwidth at the Fermi level (see also figure 2 in reference [22]).

In contrast to the case for the potential modulation [9], the differences are that (a) the Weiss oscillations in the magnetization, the susceptibility, and the specific heat (ΔC) show about a 90° phase difference between the magnetic and potential modulation cases; (b) the Weiss oscillations and SdH oscillations in ΔC are symmetric about the zero point, unlike the ones in the potential modulation case; and (c) the Weiss oscillation in ΔC has a much smaller amplitude than that in the potential modulation case (see also figure 5 of reference [9]).



(b)



(c)

Figure 3. (Continued)

3. Thermal magnetotransport coefficients

The thermal transport properties of a 2DEG in a uniform magnetic field have been extensively studied by some authors [25–33]. In this section we calculate the thermal magnetotransport coefficients of a 2DEG in a 1D periodic magnetic field in the regime of linear response theory in combination with the phenomenological transport equations for the electrical and thermal currents [25, 33].

Let $\boldsymbol{J}_e$ and $\boldsymbol{J}_Q$ be respectively the electrical and thermal current densities of a 2DEG in equilibrium. The corresponding driving forces are respectively $(-1/e)\nabla\bar{\mu}$ and $T\,\nabla(1/T)$ [25, 26, 33], where $\bar{\mu} = \mu + e\phi$, μ is the chemical potential, e the charge on the electron, and T the temperature of the system. According to Luttinger's expressions [25], the phenomenological transport equations are

$$\boldsymbol{J}_e = L_{11}\left[-\frac{1}{e}\,\nabla\bar{\mu}\right] + L_{12}\left[T\,\nabla\left(\frac{1}{T}\right)\right] \tag{7}$$

$$\boldsymbol{J}_Q = L_{21}\left[-\frac{1}{e}\,\nabla\bar{\mu}\right] + L_{22}\left[T\,\nabla\left(\frac{1}{T}\right)\right]. \tag{8}$$

Generally speaking, the phenomenological transport coefficients L_{ij} $(i, j = 1, 2)$ are tensors, i.e., $L_{ij} = (L_{ij}^{\alpha\beta})$, where $\alpha, \beta = x, y$. The transport coefficients satisfy the Onsager relation [26] $L_{ij}(\boldsymbol{B}) = L_{ji}(-\boldsymbol{B})$.

With the results of Smrčka and Středa [26], the phenomenological transport coefficients are given by the following equation:

$$L_{ij}^{\alpha\beta}(T, \mu) = \int_{-\infty}^{\infty}\left(-\frac{\partial f(\eta)}{\partial\eta}\right)\left\{[(\eta-\mu)/e]^{i+j-2}\sigma_{\alpha\beta}(\eta)\right\}\,\mathrm{d}\eta \tag{9}$$

where $f(\eta) = 1/\{1 + \exp[(\eta - \mu)/k_BT]\}$ is the Fermi–Dirac distribution function of the energy. $\sigma(\eta)$ is the zero-temperature electrical conductivity, dependent on the energy η. The above equation is valid under the assumptions of: (1) a non-interacting 2DEG; (2) elastic scattering of the electrons and the impurity particles, i.e., $\Gamma_n \ll k_BT$, where $2\Gamma_n$ is the broadening of the Landau levels due to the impurity scattering; and (3) Landau levels well separated by gaps of width proportional to $\hbar\omega_c$. With the relation $\sigma_{xy} = -\sigma_{yx}$, we know that the L_{ij} satisfy $L_{ij}^{xy} = -L_{ij}^{yx}$.

According to the definitions, the conductivity σ, the thermal transport conductivity κ, and the thermopower S are determined by [33]

$$\begin{cases} \sigma = L_{11} \\ \kappa = (L_{22} - L_{21}L_{11}^{-1}L_{12})/T \\ S = (L_{11}^{-1}L_{12})/T. \end{cases} \tag{10}$$

At zero temperature, the diagonal components of the conductivity tensors $(\sigma_{xx}, \sigma_{yy})$ are non-zero only in the energy interval $\eta \in (E_n^0 - |\varepsilon_{n,1}|, E_n^0 + |\varepsilon_{n,1}|)$ [22], where the $E_n^0 = (n + 1/2)\hbar\omega_c$ correspond to the Landau levels in the uniform magnetic field ($\boldsymbol{B}_0$). Then the diagonal components may be written as sums of the partial contributions of the individual Landau subbands [22], i.e.,

$$\begin{cases} \sigma_{\alpha\alpha}(\eta) = \sum_n\left[\sum_{k_y}\sigma_{\alpha\alpha}^{n,k_y}(\eta)\right] \\ \sigma_{\alpha\alpha}^{n,k_y}(\eta) = 0 \qquad (\eta \neq E_{n,k_y}) \\ \sigma_{\alpha\alpha}^{n,k_y}(\eta) \neq 0 \qquad (\eta = E_{n,k_y}) \end{cases} \tag{11}$$

where $\alpha = x, y$; the E_{n,k_y} are determined from equation (1). The zero-temperature Hall conductivity is

$$\begin{cases} \sigma_{yx}(\eta) = \sum_n\left[\sum_{k_y}\sigma_{yx}^{n,k_y}(\eta)\right] \\ \sigma_{yx}^{n,k_y}(\eta) = 0 \qquad (\eta \notin (E_{n,k_y}, E_{n+1,k_y})) \\ \sigma_{yx}^{n,k_y}(\eta) \neq 0 \qquad (\eta \in (E_{n,k_y}, E_{n+1,k_y})). \end{cases} \tag{12}$$

With the results of reference [22], we can write out the partial conductivity. To the order of $(\hbar\omega_1)^2$, we have

$$\begin{cases} \sigma_{xx}^{n,k_y}(\eta)|_{\text{band}} = 0 \\ \sigma_{yy}^{n,k_y}(\eta)|_{\text{band}} = 2\pi l^2 \dfrac{2\pi e^2 \tau l^2}{\hbar^2 a^2} \varepsilon_{n,1}^2 \delta(\eta - E_n^0). \end{cases} \tag{13}$$

The scattering contribution is

$$\sigma_{xx}^{n,k_y}(\eta)|_{\text{scattering}} = \sigma_{yy}^{n,k_y}(\eta)|_{\text{scattering}} = 2\pi a \frac{e^2}{h}\left(\frac{n_i V_0^2}{\pi \Gamma a}\right)[(2n+1) + B_{n,k_y}] \qquad (\eta = E_{n,k_y}) \tag{14}$$

where V_0 and n_i are respectively the potential strength and areal density of the randomly distributed impurities. We have taken $\Gamma_n = \Gamma$, independently of the Landau quantum number. B_{n,k_y} is given by

$$B_{n,k_y} = \frac{1}{2}\left(\frac{\omega_1}{\omega_c}\right)^2 u\mathrm{e}^{-u}\left[D_{n-1}^2(u) + D_n^2(u) + D_{n-1}(u)D_n(u)\right]\sin^2(Kx_0) \tag{15}$$

with

$$D_n(u) = [1 - u^{-1}]L_n^{(1)}(u) + 2L_{n-1}^{(2)}(u) \tag{16}$$

where $L_n^{(j)}(u)$ is the associated Laguerre polynomial and $x_0 = l^2 k_y$.

The partial Hall conductivity at zero temperature is given by

$$\sigma_{yx}^{n,k_y}(\eta) = \begin{cases} 0 & (\eta \notin (E_{n,k_y}, E_{n+1,k_y})) \\ 2\pi a \dfrac{2e^2}{h}\dfrac{l^2}{a}(n+1)\dfrac{1}{[1 + \lambda_n \cos(Kx_0)]^2} & (\eta \in (E_{n,k_y}, E_{n+1,k_y})) \end{cases} \tag{17}$$

where $\lambda_n = [\varepsilon_{n+1,1} - \varepsilon_{n,1}]/\hbar\omega_c$ with $\varepsilon_{n,1}$ determined by equation (1).

Substituting equations (11)–(17) into equation (9), we obtain the zero-temperature phenomenological transport coefficients $(L_{ij}^{\alpha\beta})$, to the order of $(\hbar\omega_1)^2$, as follows:

$$\begin{cases} L_{ij}^{xx}\,|_{\text{band}} = 0 \\ L_{ij}^{yy}\,|_{\text{band}} = \dfrac{2\pi e^2 \tau l^2}{\hbar^2 a^2} \displaystyle\sum_{n=0}^{\infty}\left(-\frac{\partial f(\eta)}{\partial \eta}\right)\bigg|_{\eta = E_n^0} (\varepsilon_{n,1}^2)\left[(E_n^0 - E_F)/e\right]^{i+j-2} \end{cases} \tag{18}$$

and

$$L_{ij}^{yy}|_{\text{scattering}} = L_{ij}^{xx}|_{\text{scattering}} = \frac{e^2}{h}\left(\frac{n_i V_0^2}{\pi\Gamma a}\right)\sum_{n=0}^{\infty}[(2n+1)A_{ij}^n + B_{ij}^n] \tag{19}$$

with

$$A_{ij}^n = \int_0^{a/l^2} \mathrm{d}k_y \left[-\frac{\partial f}{\partial \eta}\right]\bigg|_{\eta = E_{n,k_y}} \left[(E_{n,k_y} - E_F)/e\right]^{i+j-2} \tag{20}$$

$$B_{ij}^n = \frac{1}{2}\left(\frac{\omega_1}{\omega_c}\right)^2 u\mathrm{e}^{-u}[D_{n-1}^2(u) + D_n^2(u) + D_{n-1}(u)D_n(u)]E_{ij}^n \tag{21}$$

$$D_n(u) = [1 - u^{-1}]L_n^{(1)}(u) + 2L_{n-1}^{(2)}(u) \tag{22}$$

$$E_{ij}^n = \int_0^{a/l^2} \mathrm{d}k_y \sin^2(pKx_0)\left[-\frac{\partial f}{\partial \eta}\right]\bigg|_{\eta = E_{n,k_y}} \left[(E_{n,k_y} - E_F)/e\right]^{i+j-2}. \tag{23}$$

The non-diagonal component of $L_{ij}^{\alpha\beta}$ is

$$L_{ij}^{yx} = \frac{2e^2}{h}\frac{l^2}{a}\sum_{n=0}^{\infty}(n+1)\int_0^{a/l^2} \mathrm{d}k_y \, \frac{1}{[1+\lambda_n \cos(Kx_0)]^2} \times \int_{E_{n,k_y}}^{E_{n+1,k_y}} \mathrm{d}\eta \, \left[(E_{n,k_y} - E_F)/e\right]^{i+j-2}\left(-\frac{\partial f}{\partial \eta}\right). \tag{24}$$

Like for the conductivity tensor [22], the diagonal components of the transport coefficients are determined by

$$\begin{cases} L_{ij}^{xx} = L_{ij}^{xx}|_{\text{scattering}} \\ L_{ij}^{yy} = L_{ij}^{yy}|_{\text{scattering}} + L_{ij}^{yy}|_{\text{band}}. \end{cases} \tag{25}$$

The xx-component of the transport coefficients contains no band contribution. Due to the modulation of the magnetic field, the transport coefficients are asymmetric, i.e., $L_{ij}^{yy} \neq L_{ij}^{xx}$.

Now, on the basis of the numerical calculation, we discuss the oscillation properties of the thermopower (S), the thermal conductivity (κ), and the thermal resistance (κ^{-1}) of a 2DEG modulated by a 1D periodic magnetic field. From equation (10), we know that L_{11} is the conductivity tensor σ. Then $(L_{11})^{-1}$ is the resistivity tensor, i.e. , $\rho = \sigma^{-1} = (L_{11})^{-1}$. Its components are $\rho_{xx} = \sigma_{yy}/S_0$, $\rho_{yy} = \sigma_{xx}/S_0$, and the Hall resistivity $\rho_{xy} = \sigma_{yx}/S_0$, where $S_0 = \sigma_{xx}\sigma_{yy} + \sigma_{yx}^2$. From the results of reference [22], we know that: (1) $\sigma_{yx} \gg \sigma_{xx}$ and σ_{yy}, and (2) $|\Delta\sigma_{yx}| \ll \sigma_{yx}$. Thus we may use the approximation $S_0 \approx (\sigma_{yx})^2 \approx (n_e e/B_0)^2$ in the calculation of ρ_{xx} and ρ_{yy}, while for the Hall resistivity we use $\rho_{xy} = 1/\sigma_{yx}$ in the calculation. Therefore, the components of the thermopower are given by the following equations:

$$\begin{cases} TS_{xx} = (\sigma_{yy}/S_0)L_{12}^{xx} + (1/\sigma_{yx})L_{12}^{yx} \\ TS_{yy} = (\sigma_{xx}/S_0)L_{12}^{yy} + (1/\sigma_{yx})L_{12}^{yx} \\ TS_{xy} = (\sigma_{yy}/S_0)(-L_{12}^{yx}) + (1/\sigma_{yx})L_{12}^{yy} \\ TS_{yx} = (\sigma_{yy}/S_0)L_{12}^{yx} + (-1/\sigma_{yx})L_{12}^{xx} \end{cases} \tag{26}$$

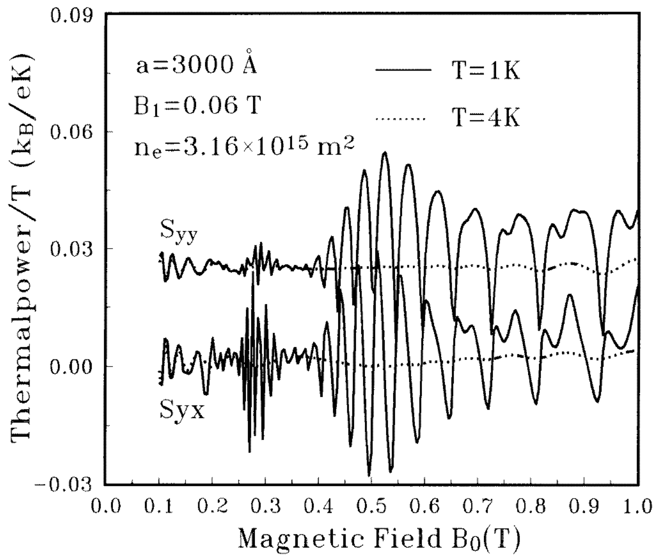
where $\sigma = L_{11}$. The components of the thermal conductivity are

$$\begin{cases} T\kappa_{xx} = L_{22}^{xx} - [L_{21}^{xx}(TS_{xx}) + (-L_{21}^{yx})(TS_{yx})] \\ T\kappa_{yy} = L_{22}^{yy} - [L_{21}^{yx}(TS_{xy}) + L_{21}^{yy}(TS_{yy})] \\ T\kappa_{yx} = L_{22}^{yx} - [L_{21}^{yx}(TS_{xx}) + L_{21}^{yy}(TS_{yx})]. \end{cases} \tag{27}$$

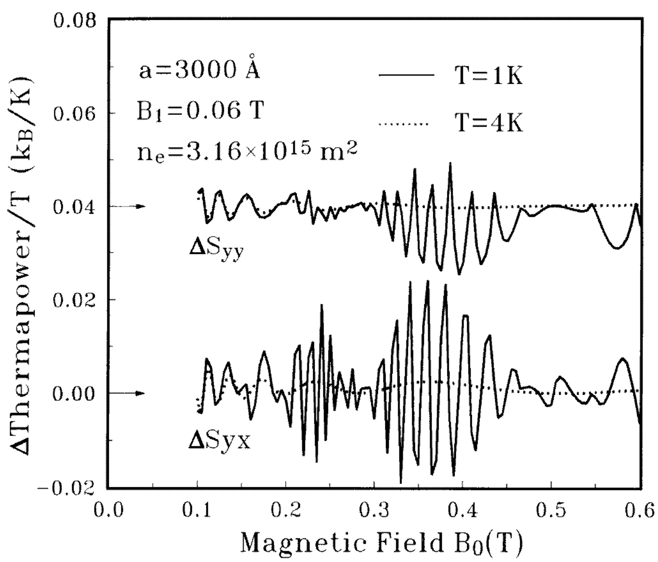
In the numerical calculation we use the following parameters: $m^* = 0.067m$, the areal carrier density $n_e = 3.16 \times 10^{15}$ m^{-2}, the electron mobility $\mu_e = 1.3 \times 10^{10}$ m^2 s^{-1}, the impurity concentration $n_i = 1 \times 10^{12}$ m^{-2}, and a small impurity broadening $\Gamma = 0.0129\sqrt{B_0}$ meV. For the 1D magnetic modulation with $a = 3000$ Å and $B_1 = 0.06$ T in the lowest-order approximation of Fourier transformation, i.e., $B_1(x) = B_1 \cos(2\pi/a)x$, the thermopower tensor divided by the temperature is plotted in figure 4(a) as a function of the uniform magnetic field B_0 in the units of $-(k_B/e)$ K. Because $S_{xx} \approx S_{yy}$, only the component S_{xx} is shown. The changes in the thermopower due to the modulating magnetic field, i.e., $\Delta S_{\mu\nu} = S_{\mu\nu}(B_1) - S_{\mu\nu}(B_1 = 0)$, are shown in figure 4(b). Different components of $\Delta S_{\mu\nu}$ have been offset, and the zero points are indicated by the corresponding horizontal arrows in figure 4(b).

From figure 4 we have the following findings.

(1) The diagonal components of the thermopower ($S_{xx} \approx S_{yy}$) are always negative, due to the negative charges of the carrier particles (electrons), while S_{xy} oscillates around zero.



(a)



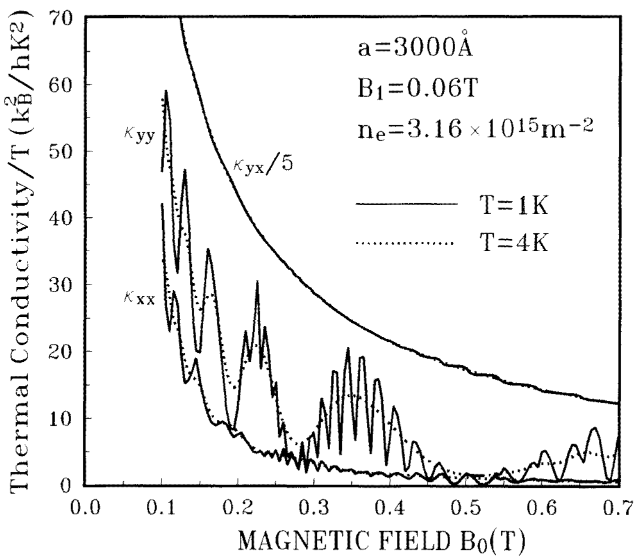
(b)

Figure 4. (a) The thermal electrical power divided by temperature versus the magnetic field B_0 for $T = 1$ and 4 K. (b) The changes in the thermal electrical power due to 1D modulation divided by temperature versus the magnetic field B_0 for $T = 1$ and 4 K.

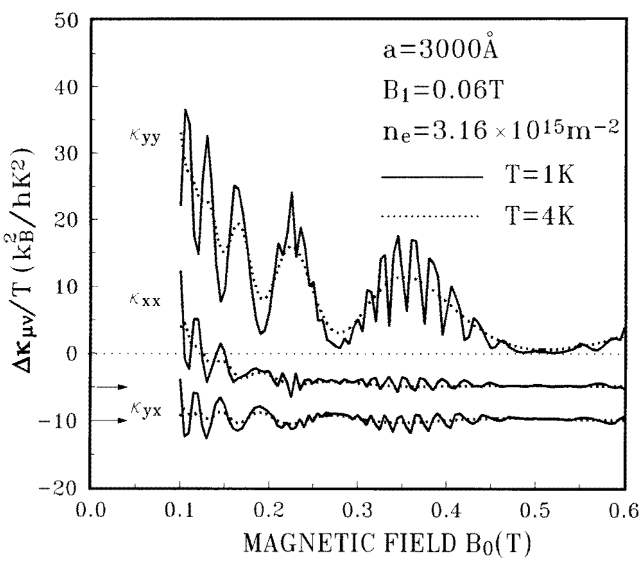
(2) At lower temperature ($T = 1$ K) and for $B_0 > 0.2$ T the SdH oscillations appear and overlap the oscillating envelope of the Weiss oscillations.

(3) The Weiss oscillations in S_{xy} (ΔS_{xy}) and S_{xx} (ΔS_{xx}) are out of phase and their SdH oscillations are 90° out of phase.

(4) The Weiss oscillations in S_{xx} (ΔS_{xx}) and ρ_{xx} ($\Delta\rho_{xx}$) are 90° out of phase, while their



(a)



(b)

Figure 5. (a) The thermal conductivity divided by temperature versus the magnetic field B_0. (b) The changes in the thermal conductivity due to the 1D periodic magnetic modulation divided by temperature for $T = 1$ and 4 K.

SdH oscillations are in phase. The Weiss oscillations in ΔS_{xy} and $\Delta\rho_{xy}$ are in phase, while their SdH oscillations are 90° out of phase (see also figures 3(a) and 3(b) in reference [22])).

(5) The Weiss oscillations in ΔS_{xy} are larger than those in the diagonal component ΔS_{xx} ($\approx\Delta S_{yy}$).

(6) At higher temperature and for a stronger magnetic field, S_{xy} tends towards zero.

We know that the results obtained here are very similar to the ones corresponding to the potential modulation case [9]. The difference is that the Weiss oscillations in S_{xy} (ΔS_{xy}) and S_{xx} (ΔS_{xx}) are out of phase for the magnetic modulation, contrary to the ones in the potential modulation case (see also figure 10 in [9]).

With the same experimental parameters as were used above, the numerical results for the thermal conductivity $\kappa_{\mu\nu}$ and the corresponding change $\Delta\kappa_{\mu\nu}$ $(= \kappa_{\mu\nu}(B_1) - \kappa_{\mu\nu}(B_1 = 0))$ due to the modulation magnetic field are shown in figures 5(a) and 5(b). All of the components are divided by the temperature so as to enable us to plot the thermal conductivity at different temperatures in the same figure. For the same reason, κ_{yx} is divided by a factor of 5, and $\Delta\kappa_{xx}$ and $\Delta\kappa_{yx}$ are moved down by five and ten units, respectively, and their zero points are indicated by the corresponding horizontal arrows. From figure 5 we find that the oscillation properties of the thermal conductivity $\kappa_{\mu\nu}$ $(\Delta\kappa_{\mu\nu})$ are similar to those of the conductivity tensor $\sigma_{\mu\nu}$ $(\Delta\sigma_{\mu\nu})$ (see also figures 2(a) and 2(b) of reference [22]). We also have the following findings.

(1) The Weiss oscillations in $\kappa_{\mu\nu}$ $(\Delta\kappa_{\mu\nu})$ have a stronger dependence on the temperature than those in $\sigma_{\mu\nu}$ $(\Delta\sigma_{\mu\nu})$. For lower magnetic fields ($B_0 < 0.2$ T), this property behaves more expectedly.

(2) The Weiss oscillations in $\kappa_{\mu\nu}$ $(\Delta\kappa_{\mu\nu})$ and $\sigma_{\mu\nu}$ $(\Delta\sigma_{\mu\nu})$ are in phase, while the corresponding SdH oscillations are 180° out of phase.

Compared with the results for the potential modulation [9], we know that (a) the value of κ_{yy} $(\Delta\kappa_{yy})$ is much larger than that of κ_{xx} $(\Delta\kappa_{xx})$; and (b) the Weiss oscillations have much larger amplitudes in κ_{yy} $(\Delta\kappa_{yy})$ than that in κ_{xx} $(\Delta\kappa_{xx})$ (see also figure 11 in [9]).

In analogy to the definition of the resistivity, we may define the thermal resistance tensor κ^{-1}, i.e., $(\kappa^{-1})\kappa = 1$. The thermal resistances multiplied by the temperatures are plotted

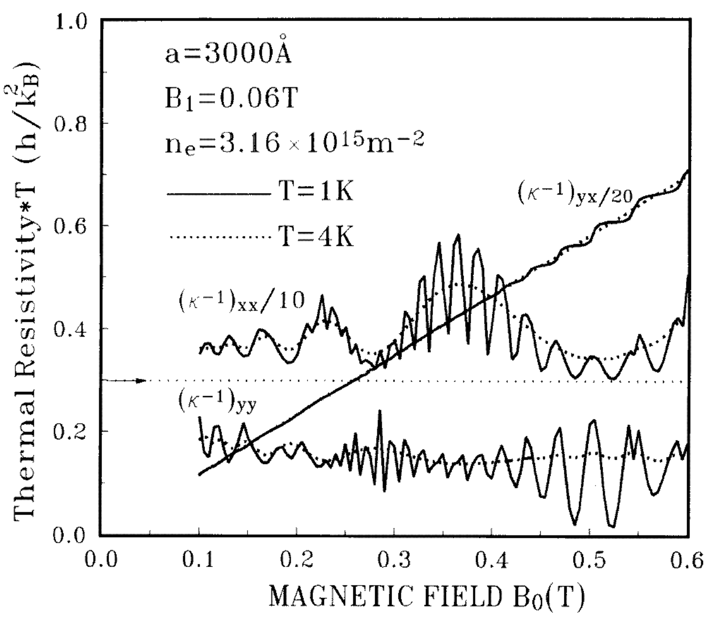


Figure 6. The thermal resistance multiplied by the temperature versus the magnetic field B_0 for $T = 1$ and 4 K.

versus the magnetic field B_0 in figure 6. The components $(\kappa^{-1})_{xx}$ and $(\kappa^{-1})_{xy}$ have been rescaled and the component $(\kappa^{-1})_{xx}/10$ has been moved upward by 0.3 units. Figure 6 shows us that the thermal resistance tensors have the following oscillation properties.

(1) At lower temperature ($T = 2$ K) and for lower magnetic fields ($B_0 < 0.4$ T), the oscillation in $(\kappa^{-1})_{xy}$ is not obvious. At $B_0 > 0.4$ T, $(\kappa^{-1})_{xy}$ has some oscillating structures.

(2) The oscillation amplitude of $(\kappa^{-1})_{xx}$ is much larger than that of the component $(\kappa^{-1})_{yy}$. This is more obvious than that in the potential modulation case [9] (see also figure 12 in [9]).

(3) The Weiss oscillations in $(\kappa^{-1})_{xx}$ and $(\kappa^{-1})_{yy}$ are out of phase while the corresponding SdH oscillations are in phase, similarly to the modulation case [9] (see also figure 12 in [9]).

Comparing to the resistivity tensor (see also figures 3(a) and 3(b) in reference [22]), we have the following findings.

(1) The Weiss oscillations in the corresponding components of the resistivity and thermal resistance are in phase, while the SdH oscillations are out of phase.

(2) The Weiss oscillations in the thermal resistance tensor $(\kappa^{-1})_{\mu\nu}$ have a stronger dependence on the temperature than those in the resistivity tensor.

At lower magnetic field, the Weiss oscillation amplitudes in the thermal resistance at the temperature $T = 4$ K are much larger than those at $T = 1$ K.

4. Conclusion

In this paper we have performed some calculations on the thermodynamics and thermal magnetotransport properties of a 2DEG modulated by a 1D periodic magnetic field similar to those carried out for the case of a potential modulation [9]. The results obtained here are similar to the ones in the potential modulation case [9], except for some essential differences as discussed above. The results show us that, because of the presence of the magnetic modulation, novel oscillations (Weiss oscillations) have been found in the thermal magnetotransport coefficients, except for the SdH oscillations, similar to those in the electrical magnetotransport coefficients [9, 16–22]. The conclusions regarding the oscillation properties are as follows.

(1) At lower temperatures ($T = 1, 2$ K), there are two kinds of oscillation in the Fermi energy and the thermal magnetotransport coefficients: the usual SdH oscillations and the Weiss oscillations. For weak magnetic fields ($B_0 < 0.2$ T), there are just Weiss oscillations because the SdH oscillations are too weak to be resolved, while at $B_0 > 0.2$ T, SdH oscillations appear and overlap the envelopes of the slow oscillations (Weiss oscillations).

(2) At higher temperatures ($T = 4, 10$ K), there are just Weiss oscillations. The Weiss oscillations have a much weaker dependence on the temperature than the sensitive dependence of the SdH oscillations.

(3) The thermal magnetotransport coefficients ($\kappa_{\mu\nu}$, $(\kappa^{-1})_{\mu\nu}$) have a stronger dependence on the temperature than the electrical ones ($\sigma_{\mu\nu}$, $\rho_{\mu\nu}$) [22].

Measurements of the thermopower of a 2DEG in a uniform magnetic field have been made by some authors [30, 32]. The results in this paper remain to be confirmed by experimental measurements.

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